Chapter 15 Problems Plus Hints
Math 2E Spring 2016

15.Review.58,59

For 58: The statement in 15.2.11 is that if f is bounded from below by m and above by M, in other words $m \leq f(x,y) \leq M$ at every point, then

$$m \cdot Area(D) \leq \iint_D f(x,y)dA \leq M \cdot Area(D) \overset{so\ we\ infer}{\iff} m \leq \frac{1}{Area(D)} \iint_D f(x,y)dA \leq M.$$

Theroem 14.7.8 states that on a closed domain D, (in other words, just take any domain and include its boundary), a continuous function will attain its minimum and maximum values, and every value in between because it is continuous. Here, m is the minimum and M is the maximum of f. And, f is continuous - so it is hopefully okay from here to finish the proof of 58.

For 59: We're to use 58, the mean value theorem. Notice $r \to 0$, the disk is shrinking. From the mean value theorem, this point (x_0, y_0) is always *inside* the domain, so it is always inside the disk D_r. As $r \to 0$, we are circling the point (a,b) so where must (x_0, y_0) go as $r \to 0$ too?

Problem 5 (and similarly 7)

The domain of integration is $[0,1] \times [0,1]$ so on this region, $xy < 1$ all the time (except at 1 point when $x = y = 1$, but this is only one point so it "doesn't affect the integral"). This is why we can use a geometric series, to write

$$\frac{1}{1-xy} = \sum_{n=0}^{\infty} (xy)^n, \quad \text{where now, show that}$$

$$\int_{x=0}^{1} \int_{y=0}^{1} \sum_{n=0}^{\infty} (xy)^n dydx = \sum_{n=0}^{\infty} \frac{1}{n^2} \quad \text{by evaluating the integral on the left side.}$$

This is equivalent to summing up the integral of all the individual terms, and note $(xy)^n = x^n y^n$,

$$\int_{x=0}^{1} \int_{y=0}^{1} \sum_{n=0}^{\infty} (xy)^n dydx = \sum_{n=0}^{\infty} \left(\int_{x=0}^{1} \int_{y=0}^{1} x^n y^n dydx \right), \quad \text{you'll want to compute this right side.}$$

Problem 8

Using the books hint, they want us to write this as $\int_{x=0}^{\infty} \frac{1}{x} \int_{y=x}^{y=\pi x} \frac{d}{dy}[\arctan(y)]\ dydx$. Then, my hint, change the order of integration to $dxdy$. (Your new bounds should be $\int_{y=0}^{\infty} \int_{x=y/\pi}^{x=y}$) and evaluate.

Problem 9

I only mentioned this one because I ran into this a ton in physics (and engineering) courses. I think it would be an awful test question because it's irrelevant to the chapter. However, I just want you to see it because you may run into, or need, these relations a lot!

The way to show these is to use the chain rule from Chapter 14.5 (In other words, that's why it wouldn't be a good 2E test question).